PSEUDO AFFINE PROJECTION ALGORITHM
NEW SOLUTION FOR ADAPTIVE IDENTIFICATION

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Abstract

In this paper, we present a new approach for adaptive echo cancellation: Pseudo Affine projection algorithm. From the original update equation of the filter coefficients of the Affine Projection algorithms, which are based on multiple dimension projection of the input signal vector, we derive a simplified solution under some realistic hypothesis. We show that our solution possesses a complexity figure close to the NLMS one but with the same convergence characteristics of the original sample-by-sample AP algorithm. Implementation details are presented in the last part of the paper.

1 Introduction

Adaptive identification of linear systems modeled by their impulse response has been studied extensively, and a number of efficient algorithms have been proposed for this purpose. Among the possible solutions, the LMS and NLMS algorithms are undoubtedly the most popular for updating the L taps of the Finite Impulse Response (FIR) filter \( H_1 \). Such algorithms provide an efficient way to implement the optimal L-samples Wiener filter that minimizes, in a stochastic approximation sense, the Mean-Square value of the filtering Error (MSE). The advantages of these algorithms are their low complexity, their simplicity of implementation and their robustness against implementation errors.

However the major drawback is their low convergence especially when colored signals such as speech excite the unknown system. To reduce this drawback, subband schemes have to be considered. Another way of meeting the constraint of convergence rate improvement for colored input signals is to use the Affine Projection Algorithm (APA) which is based on multiple dimensions projection [1]. Compared with the NLMS algorithm, APA exhibit a faster convergence rate for colored input signals but at the expense of a higher computational complexity in their basic version, even if fast solutions have recently been proposed [2]-[5].

In contrast with these approaches, the work presented in this paper describes “Pseudo AP” algorithm, which represents another efficient way to solve the issues mentioned above. The first part of the paper describes the fundamental concepts of this new approach. The main characteristics of the “Pseudo AP” algorithm are discussed (complexity figure, convergence rate) and computer simulations are given to demonstrate the usefulness of the proposed solution in the context of echo cancellation. The second part of the paper deals with implementation details of the new structure.

2 “Pseudo AP” Algorithm

In the affine projection algorithm [1] of order \( P \), the adaptive filter \( H_{t+1} \) is adjusted according to the following update equation:

\[
H_t = H_{t-1} + \mu X^*_t, p (Y_{t, p} - X^*_{t, p} H_{t-1})
\]  

(1.)

Where \( \mu \) is a parameter that lies in the range \( 0 \leq \mu \leq 1 \) and controls stability and convergence rate,

\[
Y_{t, p} = [y_t, \ldots, y_{t-p+1}]^T
\]

is a vector of the \( P \) last samples of the reference (microphone) signal, the matrix \( X_{t, p} = [X_t, \ldots, X_{t-P+1}]^T \) has dimension \( P \times L \) and consists of the \( P \) last input vectors \( X_t = [x_t, \ldots, x_{t-L+1}]^T \).

\[
X^*_t = X_t^T [X_{t, p} X_{t, p}^T]^{-1}
\]

stands for the Moore-Penrose pseudoinverse of matrix \( X_{t, p} \).

The main attractive properties of this algorithm are its good tracking behavior and its convergence rate, which is very close to that of the RLS algorithm [3]. Another important parameter is the vector \( \varepsilon_t \) of the \( P \) a posteriori errors, defined as the modeling errors that would have been produced at the same time by the filter resulting in the current weight update, i.e.:

\[
\varepsilon_t = Y_{t, p} - X^*_{t, p} H_t
\]

(2.)

Combining (1) and (2) results in the following property:

\[
\varepsilon_t = (1 - \mu) \cdot \varepsilon_t
\]

(3.)

Where \( \varepsilon_t = Y_{t, p} - X^*_{t, p} H_{t-1} \) represent the a priori error vector. Assuming in the following that \( \mu = 1 \), the relation (3) reduces to the null vector, and the update equation (1) may be reexpressed as:

\[
H_t = H_{t-1} + [X^*_t, p X^*_t, p]^{-1} X_t (Y_t - X_t^T H_{t-1})
\]

(4.)

By considering the additional assumption of stationarity of the input signal, we get the following relation between the optimal forward linear prediction coefficient vector \( A_t = [a_1, \ldots, a_{p+1}]^T \) and the energy \( E_{p+1} \) of the prediction error:

\[
A_t = \frac{1}{L} \left[ X_{t, p} X_{t, p}^T \right]^{-1} \cdot [E_{p+1}^T 0 \ldots]
\]

(5.)
On the other hand, the optimum linear predicted error vector $U_{t,P-1} = \left[ X_t, X_{t-1}, \ldots, X_{t-P+1} \right] A_P$ is related to $E_{p-1}$ through:

$$E_{p-1} = U_{t,P-1}^T X_t$$

(6.)

By substituting (5) and (6) into relation (4), we get for the update equation of the filter coefficients:

$$H_i = H_{i-1} + \frac{U_{t,P-1}}{U_{t,P-1}^T X_i} \left( y_i - X_i^T H_{i-1} \right)$$

(7.)

It has been shown that in cases where noise is added to the reference signal, the AP algorithm exhibits a large residual error. A classical solution to this problem consists in reintroducing the relaxation parameter $\mu$ as done in the relaxed affine projection given by (1). Finally, the “Pseudo AP” algorithm is defined by the following coefficient update equation:

$$H_i = H_{i-1} + \frac{\mu}{U_{t,P-1}^T X_i} \cdot U_{t,P-1} \left( y_i - X_i^T H_{i-1} \right)$$

(8.)

To optimize the computational load of the proposed algorithm, the vector $U_{t,P-1}$ consists in a tapped-delay line which stores every sample of the residual error $u_{t,P-1}$ available at the output of a forward linear prediction filter of order $P-1$.

3 Experimental results

We will now demonstrate the usefulness of the proposed approach in the context of acoustic echo cancellation. In the following experiments, conducted with synthetic signals, the system impulse response was measured in a car between the two transducers of a hands-free system. This impulse response was truncated to 256 samples (for a sampling frequency of 8 kHz). For this truncated impulse response, an identification filter of 256 taps will produce an exact matching. In the absence of any a priori knowledge of the optimal filter, we have taken the initial value $H_{-1} = 0$ for the filter coefficients. In all experiments, the step-size of the algorithms is set to the same value $\mu = 0.9$ and the projection order of the AP algorithms (original and “Pseudo”) is set to $P = 9$. For the excitation process, we will first consider the case of a “USASI” input sequence.

Synthetic excitation

The learning curves of the NLMS and “Pseudo AP” algorithms are given in Fig. 1. Also shown on this figure are the time evolutions of the energy of the microphone signal (i.e. the acoustic echo) and of the measurement noise. We can see that the simulations reveal the big advantage in convergence rate of the “Pseudo AP” algorithm over the NLMS algorithm. It should be noted that this faster convergence rate is gained at the expense of a higher steady-state MSE.

![Figure 1. Learning curves of the NLMS and “Pseudo AP” algorithms. Mean-Square error convergence for “USASI” sequence](image1)

The behavior of the learning curves of “Pseudo AP” and AP algorithms is illustrated in Fig. 2. For “USASI” input sequences, we may notice that the “Pseudo AP” version exhibits a slight disadvantage in convergence speed in comparison with the original AP algorithm. However, after the initial convergence period, we see that the steady-state MSE reaches the same level for the two algorithms.

Change in the echo path

The comparative test in a time-varying context is of great practical interest since convergence and tracking are two different entities. Figs. 3 and 4 refer to the learning curves of the NLMS, original AP and “Pseudo AP” algorithms for an artificially time-varied echo path; in fact the fine structure of the echo path remains the same as before excepted that a linear increase in the loudspeaker gain has been introduced into the path. It is readily observed in Fig. 3 that the tracking performance of the “Pseudo AP” is better than the NLMS one. Note also that the misadjustment occurring during the echo path variations is decreased when using the “Pseudo AP”. If we concentrate now on the tracking
capabilities of the original AP and “Pseudo AP” algorithms (see Fig. 4), we can see that these two solutions have exactly the same behavior (misadjustment and tracking capabilities) during the echo path variations.

![Figure 3](image1.png)

Figure 3. Learning curves of the NLMS and « Pseudo AP » algorithms under a sudden change in the echo path. Mean-Square error convergence for « USASI » sequence

The conclusion we can draw from the above considerations is that the “Pseudo AP” algorithm catches the main attractive properties of the original APA, i.e. fast convergence rate and good tracking ability.

**Real Speech excitation**

The learning curves of the previous algorithms for realistic echo signals produced by speech inputs are displayed in Figs. 5 and 6. One can easily observe i) the enhanced performance of the “Pseudo AP” algorithm compared with the NLMS and ii) the close behavior obtained in the simulations between the original and “Pseudo AP” algorithms.

Some interesting conclusions can be drawn by a closer look at the curves. One can see for example in Fig. 6 that the initial convergence curves of the “Pseudo AP” and original AP are much closer than those displayed in Fig. 2 with the synthetic “USASI” input signal. Moreover, focusing on the steady-state behavior of these two algorithms, one can see in Fig. 6 that the two curves have approximately the same time variations. This means that the “Pseudo AP” algorithm behaves as the original AP of the same projection order on real speech inputs.

![Figure 5](image2.png)

Figure 5. Learning curves of the NLMS and « Pseudo AP » algorithms Mean-Square error convergence for speech sequence

![Figure 6](image3.png)

Figure 6. Learning curves of the « Pseudo AP » and original AP algorithms. Mean-Square error convergence for speech sequence.

## 4 Implementation details

In the previous sections, simulations on both artificially and real-world signals were used to demonstrate the main attractive properties of the “Pseudo AP”. In this section, we will focus on some implementation details of the proposed approach.

As it has already been pointed out in section 1, the implementation is divided into two separate phases. The first stage corresponds to a classical linear prediction scheme, which produces, at time \( t \), the residual error \( u_t \) which results from the prediction of \( x_t \) using linear
combinations of data values \( x_{t-1}, x_{t-2}, \ldots, x_{t-P+1} \). Secondly, a filtering and adaptation phase which produces the \textit{a priori} error and updates the filter coefficients from the knowledge of the decorrelated signal \( u_{t-p+1} \).

To compute the prediction coefficients \( A_p \) as function of the correlation statistics of the input signal we have chosen the lattice filter structure based on the minimization of the MSE over a block size of \( N \) samples. The values of the optimal prediction coefficients \( A_p \) are updated every \( N/K \) samples. In the previous sections, we have used for all simulations the following parameters: \( N = 160 \) (block length of 20 ms), \( K = 4 \), and the optimal prediction coefficients \( A_p \) are computed with the Levinson-Durbin recursion.

![Figure 7. Implementation of the “pseudo AP”](image)

Figure 7. Implementation of the “pseudo AP”.

To further improve the robustness of the proposed solution, we propose to take advantage of the lattice structure. This choice enables the coefficients at a given stage to be computed independently of those following that stage. Thus, optimum values can be computed successively along the structure without affecting other coefficients. To continuously estimate the optimum order \( P_{opt} \), we have analyzed two main approaches [6]: the AIC (for An Information-theoretic Criterion) and the FPE (for the Final Prediction Error). For these two criteria, the optimum prediction order \( P_{opt} \) is given by the minimum of the criteria:

\[
P_{opt} = \text{ArgMin}_{P \in \{1, \ldots, P_{max}\}} \{ \text{FPE}(P) \text{ or } \text{AIC}(P) \} \quad (9.1)
\]

Our experiments on real speech sentences indicate that the AIC and FPE approaches achieve approximately the same prediction gains.

As a final remark, we may note that the implementation structure proposed in Fig. 7 is very similar to the structure proposed in [7,8] for LMS coupled adaptive prediction and system identification. In fact, to prevent ill-conditioning in the input covariance matrix, a prewhitening adaptive filter is used to reduce the eigenvalue spread. However, the previous works reported that a very strong coupling effect was between the update equations of the two adaptive filters (the prediction and the identification filters). This coupling effect is at the origin of the instability reported by the previous authors as soon as the prediction order was greater than \( P = 4 \).

In contrast with these works, the technique presented in this paper offers the combined advantages of relatively low complexity, stability for a prediction order \( P \) greater than 4 and for step-sizes \( \mu \) ranging from a value of 0 to 1, fast convergence and low memory requirements. Moreover, in applications where the acoustic echo control system is implemented on the same DSP as a low-rate parametric speech codec, the short-term and long-term predictors of the speech decoder should be efficiently reuse to improve the complexity figure. In this special case, the computational complexity of the “Pseudo AP” algorithm is the same as the NLMS one. All these properties demonstrate the computational efficiency and the practical interest of the proposed approach.

5 Conclusions

This paper has introduced a new approach for adaptive identification of time-variant impulse responses. Simulations made on synthetic and real-world signals demonstrate that the “Pseudo AP” has the same behavior (convergence rate, steady state) as the original sample-by-sample projection algorithm. Moreover, the proposed approach has exactly the same tracking behavior as the original APA. Finally, implementation details are given that demonstrate the usefulness of the approach and the simplicity of the implementation.

REFERENCES


